UNIVERSITY OF MINES AND TECHNOLOGY (UMAT)

Tarkwa, Ghana

DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

NUMERICAL ANALYSIS REPORT

QUADRATIC INTERPOLATION

COURSE CODE

CE363

CLASS

CE3A

GROUP

TWELVE(12)

Interpolation: Why Quadratic?

What is Interpolation?

The process of estimating intermediate values between precise, known data points. It assumes a relationship exists between points to predict missing data.

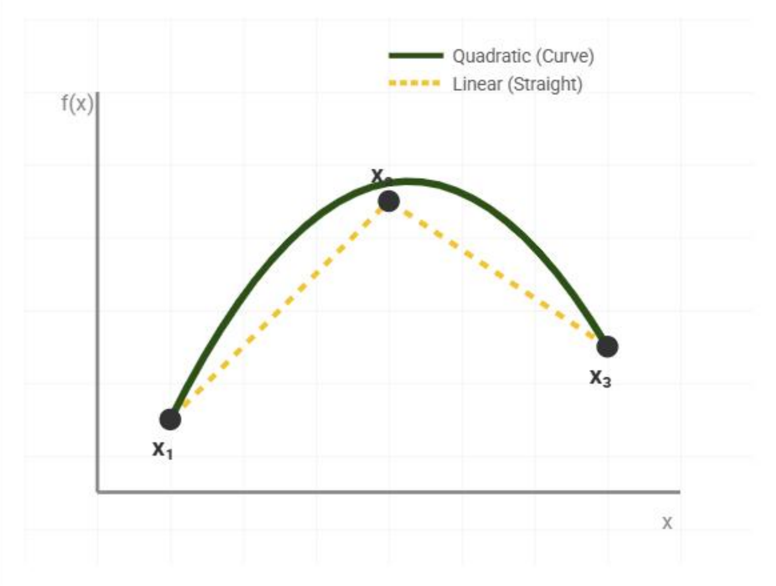
Polynomial Basis

- **General Rule:** To fit **n** data points, we generally find a unique polynomial of order **(n-1)**.
- **Linear (2 points):** Creates a straight line (1st order). Fast, but assumes constant rate of change.
- **Quadratic (3 points):** Creates a parabola (2nd order). Captures curvature and changing slopes.

When to use Quadratic?

Use when data follows a curved trend (non-linear) and you have at least three points. It offers significantly better accuracy than linear methods for physics and engineering problems.

LINEAR VS. QUADRATIC FIT



Linear connects dots blindly.
Quadratic fits the flow.

Quadratic Interpolation Formula

THE GENERAL EQUATION (NEWTON FORM)

$$f(x) = b_1 + b_2(x - x_1) + b_3(x - x_1)(x - x_2)$$

$$b_1 = f(x_1)$$

START POINT

Value at the first data point

FIRST DIVIDED DIFFERENCE

Slope between first two points (Linear part)

$$b2 = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

SECOND DIVIDED DIFFERENCE

Adds curvature (Parabolic component)

$$b3 = \frac{\dfrac{f(x_3) - f(x_2)}{x_3 - x_2} - \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}}{x_3 - x_1}$$

Note: b_3 represents the rate of change of the slope.

This formulation is numerically stable and easy to compute recursively.

PROCESS METHODOLOGY

Step-by-Step Method

1

Select Points

Identify three data points (x_1, x_2, x_3) closest to the target x. Preferably, x should be within the range.

2

Compute Coeffs

Calculate b_1, b_2, b_3 using the Newton divided-difference formulas derived from the data.

3

Evaluate f(x)

Substitute the target x and coefficients into the quadratic equation to find the estimate.

4

CHECK REASONABLENESS

Ensure the result lies within the expected range of neighbors. Large deviations may suggest oscillation (Runge's phenomenon).

5

ALTERNATIVES?

If accuracy is insufficient or spacing is uneven, consider Spline Interpolation.

Pro Tip: For best accuracy, choose points such that $x_1 < x < x_3$, keeping x centered in the interval.

PRACTICAL APPLICATION

Example 1: Car Position

Problem

A car travels along a straight road. Find the position at **t = 3s** given the following data points:

t=0, x=0	t=2, x=10	t=4, x=36
(0, 0)	**(2, 10)**	**(4, 36)**

SOLUTION STEPS

1. CALCULATE COEFFICIENTS

$b_1 = f(x_1) = 0$

$b_2 = (10 - 0) / (2 - 0) = 5$

$b_3 = [(36-10)/(4-2) - 5] / (4-0)$

$= (13 - 5) / 4 = 8 / 4 = 2$

2. SUBSTITUTE INTO FORMULA

$f(x) = b_1 + b_2(x-x_1) + b_3(x-x_1)(x-x_2)$

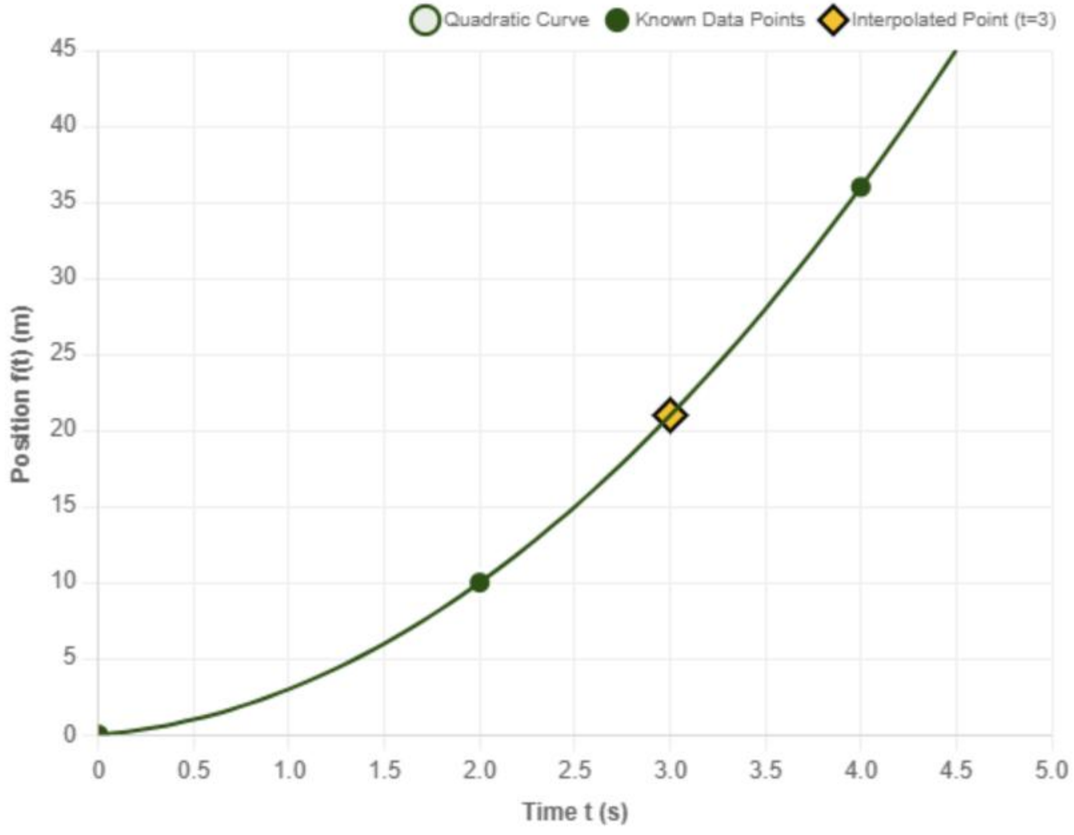
$f(3) = 0 + 5(3-0) + 2(3-0)(3-2)$

FINAL CALCULATION

$f(3) = 15 + 6 = 21$ m

Position vs. Time Graph

$f(t) = 2t^2 + t$



The quadratic curve (green line) smoothly connects the three known points. The red point indicates our interpolated result at t=3s.

DATA ANALYSIS EXAMPLE

Example 2: Rocket Velocity

Given Data

Target t = 16s

Time t (s)	Velocity v(t) (m/s)
0	0
10 (x_1)	**227.04**
15 (x_2)	**362.78**
20 (x_3)	**517.35**
22.5	602.97
30	901.67

** Highlighted points selected for interpolation near t=16s*

COMPUTATION SUMMARY

$b_1 = 227.04$

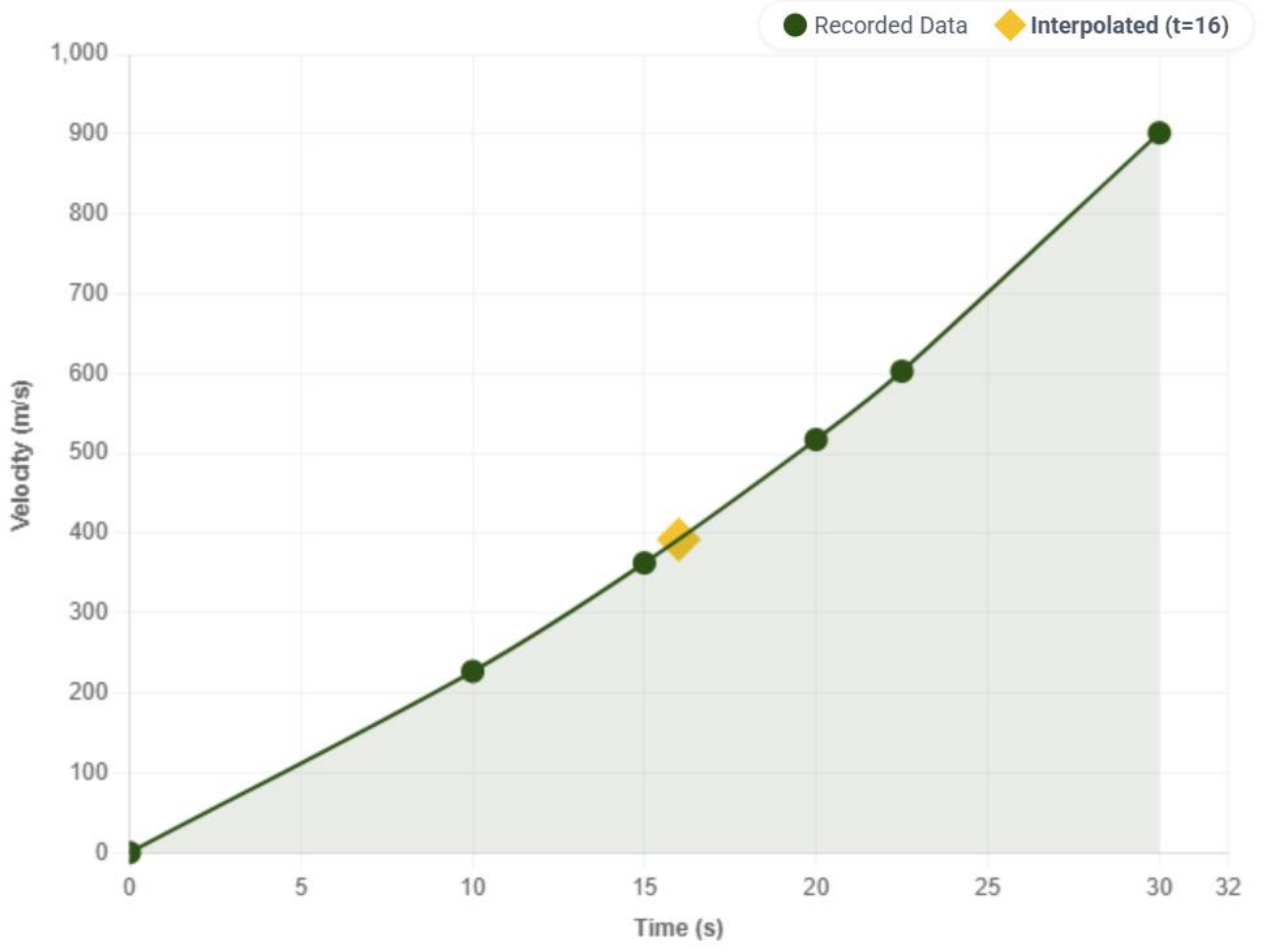
$b_2 = (362.78 - 227.04)/5 = 27.148$

$b_3 = (30.914 - 27.148)/10 = 0.3766$

Substituting into quadratic formula:

$v(16) = b_1 + b_2(6) + b_3(6)(1)$

$v(16) \approx 392.19$ m/s



The chart visualizes the upward velocity trend. The quadratic fit (green line) effectively captures the non-linear acceleration of the rocket between the sampled points.

REAL-WORLD USES

Applications of Quadratic Interpolation

Physics & Mechanics

Ideal for projectile motion problems (parabolic trajectories). Estimates displacement, velocity, or acceleration from sparse time-series data.

Engineering Design

Predicts material properties (e.g., thermal expansion, stress-strain) and calibrates sensors by fitting curves to experimental measurements.

Computer Graphics

Fundamental to quadratic Bézier curves used in vector graphics and font rendering. Smoothens animation between keyframes.

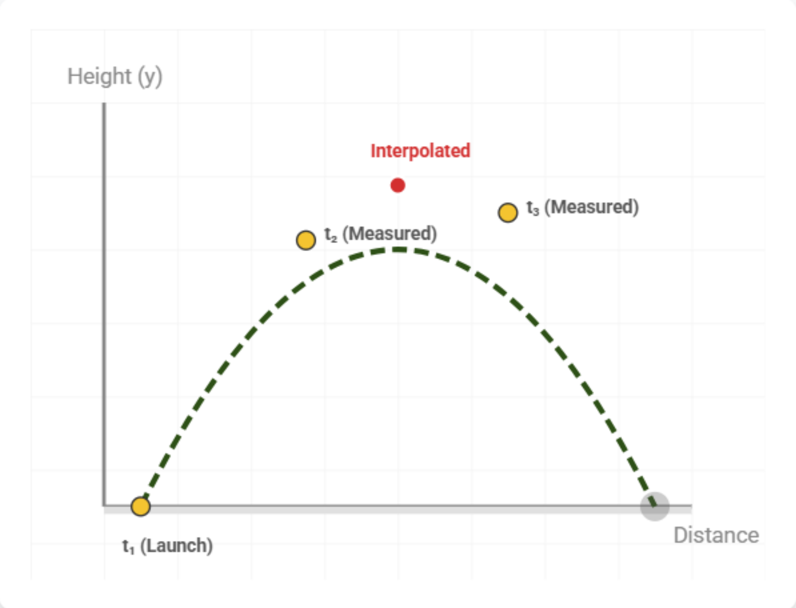
Economics & Finance

Estimates short-term trends in volatile markets where linear assumptions fail. Models non-linear growth patterns like GDP or interest rates.

Scientific Computing

Acts as a building block for Simpson's 1/3 Rule (numerical integration) and optimization algorithms involving parabolic approximation.

PHYSICS CASE: PROJECTILE MOTION



By measuring just 3 positions, quadratic interpolation can accurately reconstruct the entire flight path of a projectile.

CRITICAL ANALYSIS

Advantages & Limitations

Advantages

Superior Accuracy
Significantly more accurate than linear interpolation for non-linear data by utilizing three points instead of two.

Captures Curvature
Detects turning points and models curved trends (parabolic behavior) that straight lines miss completely.

Simplicity
Relies on elementary arithmetic and algebraic substitution. Computationally inexpensive compared to splines.

Foundational Method
Serves as the essential stepping stone for understanding higher-order polynomials and Simpson's rules.

Limitations

Data Requirement
Strictly requires at least three distinct data points. Cannot be applied if only two points are available.

Sensitivity to Noise
Errors in input data are amplified. A single bad point distorts the entire parabola significantly.

Runge's Phenomenon
Can oscillate wildly over large intervals or with uneven spacing, leading to poor approximations at the edges.

Complex Systems
Not suitable for highly complex or erratic data with multiple inflection points (requires splines).

Balance trade-offs: Choose quadratic when data is smooth and locally parabolic.

METHOD SELECTION GUIDE

Comparative Analysis

CRITERIA	LINEAR	QUADRATIC	CUBIC SPLINE
Points Used	**2 Points** (Neighboring)	**3 Points** (Local Group)	**All Points** (Whole Dataset)
Degree (n)	1st Order Straight Line	2nd Order Parabola	3rd Order Piecewise Cubic
Smoothness	**C^0 Continuity** Sharp corners at points	**Local Curvature** Smooth in interval	**C^2 Continuity** Globally smooth
Strengths	✓ Simple & Fast ✓ Always stable	✓ Captures peaks ✓ Better accuracy	✓ Very smooth ✓ No sharp edges
Best For	Dense data tables, quick lookups	**Short intervals, mild non-linearity**	Graphics, trajectory design, complex curves
Cautions	High error on curves	Oscillation risk	Complex math

Basic Method | **Presentation Focus** | Advanced Method

PRESENTATION TEAM

Group Members

COURSE CODE

CE363

Numerical Analysis

CLASS

CE3A

GROUP

TWELVE(12)

DEPT. OF COMPUTER SCIENCE AND ENGINEERING

GC	GBANG-DERY CHARLES	FCM.41.008.131.23
FP	FOSU PRINCE	FCM.41.008.127.23
EP	ESHUN PRINCE OFORI	FCM.41.008.124.23
EE	ERIC EHOMAH	FCM.41.008.123.23
FS	FOSU SAMUEL ELLORM	FCM.41.008.128.23

Thank you for your attention.